Sometimes students come up looking for explanations of concepts they come across in books. Actually, in Quant, you can establish innumerable inferences from the theory of any topic. The point is that you should be comfortable with the theory. You should be able to deduce your own inferences from your understanding of the topic. If you come across some so-called rules, you should be able to say why they hold. Let's discuss a couple of such rules from number properties regarding GCF (greatest common factor). Many of you might read them for the first time. Stop and think why they must hold.

Rule 1: Consecutive multiples of 'x' have a GCF of 'x'

Explanation: What do we mean by consecutive multiples of x? They are the consecutive terms in the multiplication table of x. For example, 4x and 5x are consecutive multiples of x. So are 18x and 19x…

What will be the greatest common factor of 18x and 19x? We know that x is their common factor. Do 18 and 19 have any common factors (except 1)? No. So greatest common factor will be x. Take any two consecutive numbers. They will have no common factors except 1. Hence, if we have two consecutive factors of x, their GCF will always be x.

For more on common factors of consecutive numbers, check:

http://www.veritasprep.com/blog/2011/09 … c-or-math/

http://www.veritasprep.com/blog/2011/09 … h-part-ii/

Can you derive some of your own 'rules' based on this now? Let's give you some ideas:

Two consecutive integers have GCF of 1.

Two consecutive odd multiples of x have GCF of x.

Rule 2: The G.C.F of two distinct numbers cannot be larger than the difference between the two numbers.

Explanation: GCF is a factor of both the numbers. Say, the GCF of two distinct numbers is x. This means the two numbers are mx and nx where m and n have no common factor. What can be the smallest difference between m and n? m and n cannot be equal since the numbers are distinct. The smallest difference between them can be 1 i.e. they can be consecutive numbers. In that case, the difference between mx and nx will be x which is equal to the GCF. If m and n are not consecutive integers, the difference between them will be much larger than x. The difference between mx and nx cannot be less than x.

Say, GCF of two numbers is 6. The numbers can be 12 and 18 (GCF = 6) or 12 and 30 (GCF = 6) etc but they cannot be 12 and 16 since both numbers must have 6 as a factor. So after a multiple of 6, the other multiple of 6 must be at least 6 away.

Let's look at a question based on these concepts now.

Question 1: What is the greatest common factor of x and y?

Statement 1: Both x and y are divisible by 4.

Statement 2: $x - y = 4$

Solution:

Statement 1: Both x and y are divisible by 4

We know that 4 is a factor of both x and y. But is it the highest common factor? We do not know. There could be another factor common between x and y and hence highest common factor could be greater than 4. e.g. 4 and 16 have 4 as the highest common factor but 12 and 36 have 12 as the highest common factor though both pairs have 4 as a common factor.

Statement 2: $x - y = 4$

We know that x and y differ by 4. So their GCF cannot be greater than 4 (as discussed above). The GCF could be any of 1/2/4 e.g. 7 and 11 have GCF of 1 while 2 and 6 have GCF of 2.

Taking both statements together: From statement 1, we know that x and y have 4 as a common factor. From statement 2, we know that x and y have one of 1/2/4 as highest common factor. Hence 4 is the highest common factor.

Answer (C)

We say goodbye to GMAT coordinate geometry for a while now and start today's post with some magic tricks:

<u>Trick 1</u>

Step 1: Pick any two consecutive integers (Don't tell me what they are!).

Step 2: Multiply them.

Whatever your numbers, the product is an even number! If you are wondering how I knew that, you desperately need to read this post. If you are shaking your head in disappointment, wait, I have more.

Note: 0 is even. So are -2, -4, -6 etc

<u>Trick 2</u>

Step 1: Pick any three consecutive integers.

Step 2: Multiply them.

Whatever your numbers, the product is a multiple of 6. Now, if you are surprised, great, go ahead and read this post. If you are still bewildered that why the heck am I calling 'simple Math' 'magic tricks', wait, give me one last chance.

<u>Trick 3</u>

Step 1: Pick any four consecutive integers.

Step 2: Multiply them.

Whatever your numbers, the product is definitely a multiple of 24. I hope I have caught your interest now. If you are still not surprised, tell me the greatest number which is definitely a factor of the product of any five consecutive integers. If you do get an answer, scroll down to the bottom of the post to check it. If you get the correct answer, you don't need to read this post. Hopefully, I will see you next week with something more challenging. If not, then stick around.

So how did I know the numbers which were definitely factors of consecutive integers? Let me explain you the simple Math involved.

Theory:

Pick any two consecutive numbers. e.g. (5, 6) or (101, 102) or (999, 1000) etc ..

Do you agree that one of them, and only one of them will be even? Every alternate number has 2 as a factor so no matter which two consecutive numbers you pick, one of them will definitely have 2 as a factor and the other will not.

Notice a few things about integers:
…-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16…

- Every number is a multiple of 1
- Every second number is a multiple of 2
- Every third number is a multiple of 3
- Every fourth number is a multiple of 4 and so on…

If we pick any 2 consecutive integers, one and only one of them will be a multiple of 2: e.g. pick 4, 5 (4 is a multiple of 2) or pick 11, 12 (12 is a multiple of 2) etc.

If we pick any 3 consecutive integers, at least one of them will be a multiple of 2 e.g. pick 3, 4 and 5 (4 is a multiple of 2) or pick 6, 7 and 8 (6 and 8 are multiples of 2) etc. Of the 3 numbers, exactly one will be a multiple of 3 e.g. pick 3, 4 and 5 (3 is a multiple of 3) or pick 6, 7 and 8 (6 is a multiple of 3) etc. Hence the product of the 3 consecutive integers will be a multiple of 2 and 3 and therefore, of 6.

If we pick any 4 consecutive integers, two of them will be multiples of 2 e.g. pick 3, 4, 5 and 6 (4 and 6 are multiples of 2) or pick 6, 7, 8 and 9 (6 and 8 are multiples of 2) etc. Of the 4 numbers, at least one will be a multiple of 3 e.g. pick 3, 4, 5 and 6 (3 and 6 are multiples of 3) or pick 5, 6, 7 and 8 (6 is a multiple of 3). Also, exactly one number will be a multiple of 4 e.g. pick 3, 4, 5 and 6 (4 is a multiple of 4) or pick 5, 6, 7 and 8 (8 is a multiple of 4) etc. Hence the product will be a multiple of at least 4, 3 and 2 (from the even integer which is not a multiple of 4) and therefore, of 24.

Hope the concept is clear to you. Let’s look at a question now.

Question: Given that n is any integer such that (n-1)*n*(n+1) is divisible by 24, which is the following must be true?

1.Either n is divisible by 8 or (n+1) is divisible by 4

2.Either n or (n^2 – 1) is divisible by 3

3.n is not divisible by 16

Solution:

(n-1), n and (n+1) are consecutive integers. Either (n-1) and (n+1) are even or n is even. We know that n has 8 and 3 as factors. If n is even, it will be a multiple of 8 since (n-1) and (n+1) will be odd. If (n-1) and (n+1) are even, at least one of them will be a multiple of 4 (to get 8 as the factor of their product). Either (n-1) or (n+1) could be the integer divisible by 4. Hence statement I is not necessarily true.

Since the product is divisible by 24, one of (n-1), n and (n+1) must be divisible by 3. Either n or (n-1)(n+1) = (n^2 – 1) must be divisible by 3. Hence statement II must be true.

If the three consecutive integers are 15, 16 and 17, n could be divisible by 16. All we know is that the product is divisible by 24, it could be divisible by 48 or higher numbers. So statement 3 is not necessarily true.

Answer to the 5 consecutive integers question: 120 (Shortcut – If you need 5 consecutive integers, take 1, 2, 3, 4 and 5 – their product is 120. This will be the maximum number which will definitely be a factor of any five consecutive integers.)

Today we will continue from where we left our last post. In the last post, we discussed that of any two consecutive integers, one and only one of them will be even. Out of 20 and 21, only 20 is even. Since 2 is a factor of 20, it will not be a factor of 21. Does that make sense? Sure. Every second number will have 2 as a factor.

On the same lines, can both the consecutive numbers have 3 as a factor? Let's take the same example – 20 and 21. 3 is a factor of 21. Can it be a factor of 20 as well? Do we even need to check? Since 21 is a multiple of 3, the previous multiple must be 3 places before (i.e., 18) and the next multiple of 3 must be 3 places ahead (i.e., 24).

What do you conclude then? Two consecutive integers can only have 1 common factor and that is 1. This means that if we pick any two consecutive integers, they will have no common factor other than 1. Say if 5 were their common factor, the numbers would be 5/10/15… apart e.g. 25 and 30. They cannot be consecutive. If 11 were their common factor, the numbers would be at least 11 apart e.g. 11 and 22. They cannot be consecutive.

Out of three consecutive integers, two could have 2 as a common factor e.g. 20, 21 and 22. Both 20 and 22 have 2 as a common factor. But can 3 be a common factor of any two numbers? No. One and only one number will be a multiple of 3.

Another way to look at this – Say we have the following consecutive integers:

(N – 4), (N – 3), (N – 2), (N – 1), N, (N + 1), (N + 2), (N + 3), (N + 4)

We are given that 2, 3, 5 and 7 are factors of N. What can we say about the factors of the rest of the numbers?

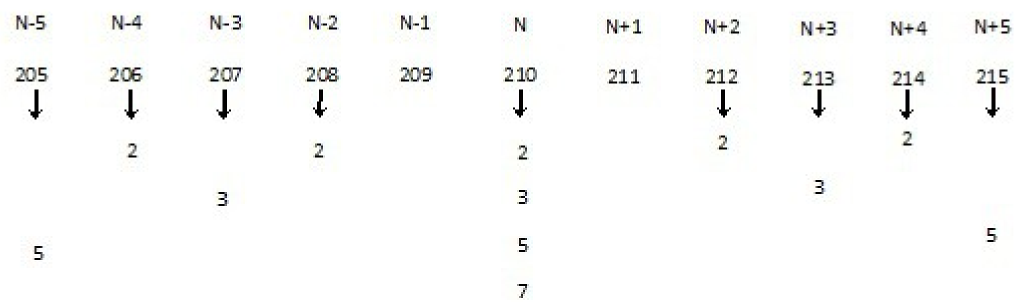
1.(N + 1) and (N – 1) both will NOT have any of 2, 3, 5 and 7 as factors. They are consecutive with N. If N is a multiple of 2, 3, 5 and 7, the next multiples of these numbers will be farther away.

2.(N + 2) and (N – 2) both will have 2 as a factor. They are 2 steps away from N. Since N is a multiple of 2, they will be multiples of 2 too. (N + 2) and (N – 2) both will NOT have 3, 5 and 7 as factors. (N + 2) and (N – 2) are only two steps away from N. The next multiples of 3, 5 and 7 will be farther away.

3.(N + 3) and (N – 3) will have 3 as a factor. They both are 3 steps away from N. Since 3 is a factor of N, it will also be a factor of these two numbers. They will not have 2, 5 and 7 as factors.

4.(N + 4) and (N – 4) will have 2 as a factor. They are 4 steps away from N. Since N is a multiple of 2, they will be multiples of 2 too. But, they will NOT have 3, 5 and 7 as factors.

The diagram given below will help you visualize this concept.



Now think: If you pick any two consecutive integers, can they both have 4 as a factor? or 7 as a factor? or 99 as a factor? No! Once you get one multiple of 99, you will not get another one in the next 98 numbers. The next multiple will appear when you add 99 to this multiple. For example, say you pick 99. Can 100, 101, 102… be multiples of 99? No. The next multiple of 99 will be 198. Therefore, numbers from 100 to 197 will not be multiples of 99.

We can say that consecutive numbers will not have any common factor other than 1. (1 is a factor of every number.)

Let’s look at how knowing this property can be useful.

Question: For every positive even integer n, the function f(n) is defined to be the product of all the even integers from 2 to n, inclusive. If p is the smallest prime factor of f(100) +1, then p is

(A) between 2 and 20
(B) between 10 and 20
(C) between 20 and 30
(D) between 30 and 40
(E) greater than 40

Solution:

First of all, the question sounds much more complicated than it actually is. Just put some values for n and try and figure out what the function looks like.

f(2) = 2

f(4) = 2*4

f(6) = 2*4*6

and so on…

f(100) = 2*4*6*…*98*99*100 = (2*1)*(2*2)*(2*3)*…*(2*48)*(2*49)*(2*50)

f(100) = (2^50) * 1*2*3*…*48*49*50

Can I say that all numbers from 2 to 50 are definitely factors of f(100)? Sure. We can see above that they are. Then what can I say about the factors of f(100) + 1? Since f(100) and f(100) + 1 are consecutive integers, can I say that they share only one common factor and that is 1? Yes, I can. We just saw this concept above. This means that f(100) + 1 will not have any factor lying between 2 and 50, inclusive because each of these numbers is a factor of f(100). So if p is a prime factor of f(100) + 1, can we say that it must be greater than 50? Yes, we can. We know that p cannot be 1 since p is a prime factor. The next factor of f(100) + 1 must be greater than 50. Since it is greater than 50, it will definitely be greater than 40 too.

Answer (E)

I think you will agree that the solution is much simpler than what you would have first expected. This is an important number property and could be useful in a range of situations. Make sure you understand it well, and as always, keep practicing!